Advanced Modulation/Synchronization/Coding Schemes

Parameter Estimation and Synchronization in Digital Receivers

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Introduction

- ***Turbo codes***
  - Berrou et al. (ICC'93)
  - Shannon bound (1948)
  - Computational complexity

- ***Demodulator***
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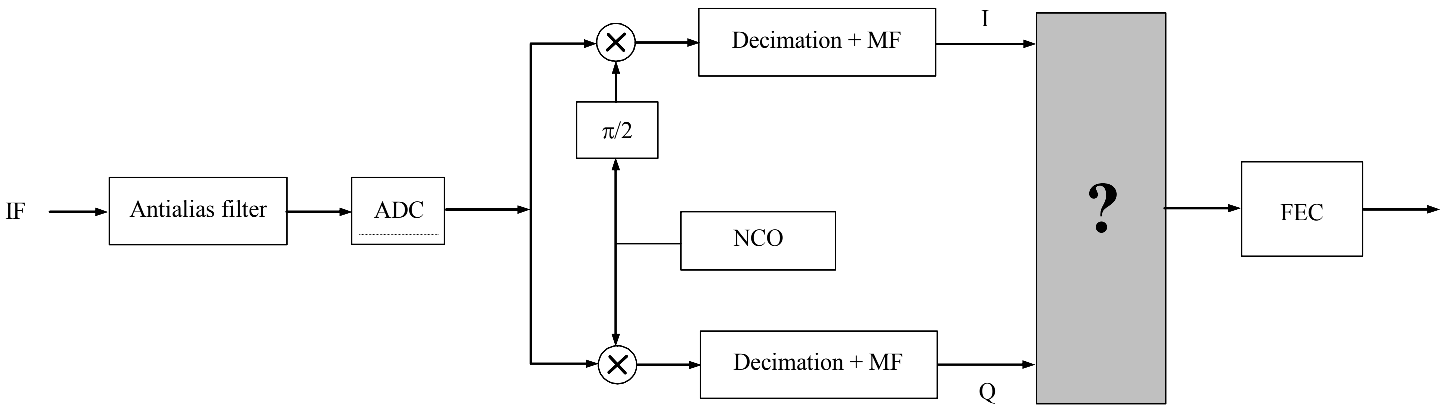
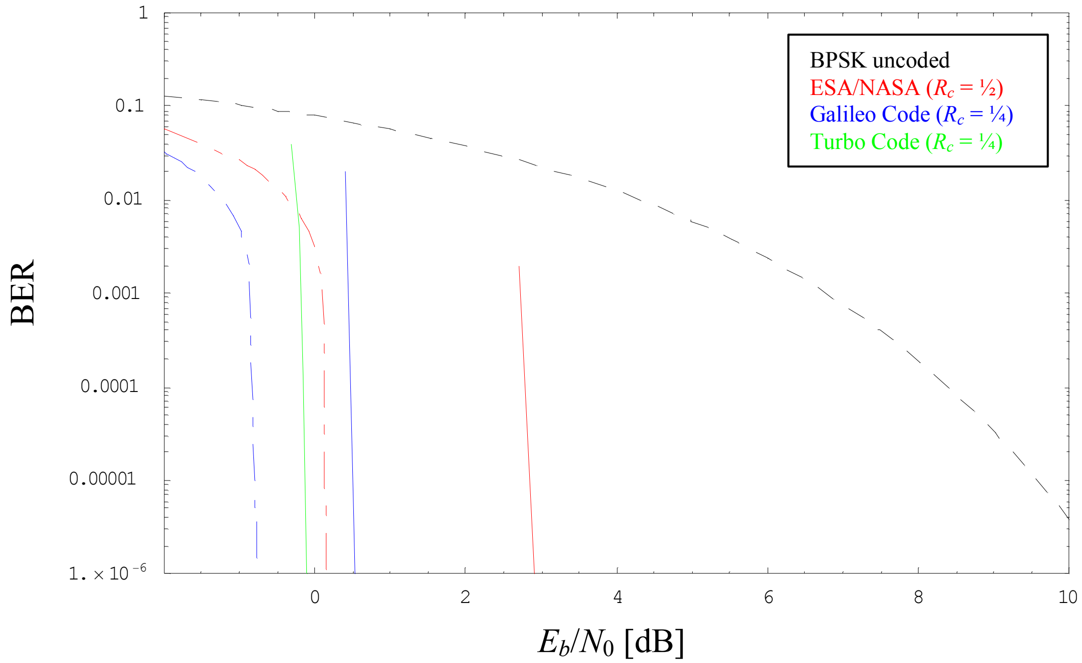
    - Continuous vs. burst mode
    - Impact of phase noise (small/large bandwidths)
  - Challenging operational conditions: very low SNR values, burst mode
  - Major parameters to be estimated: carrier frequency/phase, symbol timing

Introduction



Basic receiver architecture

Introduction

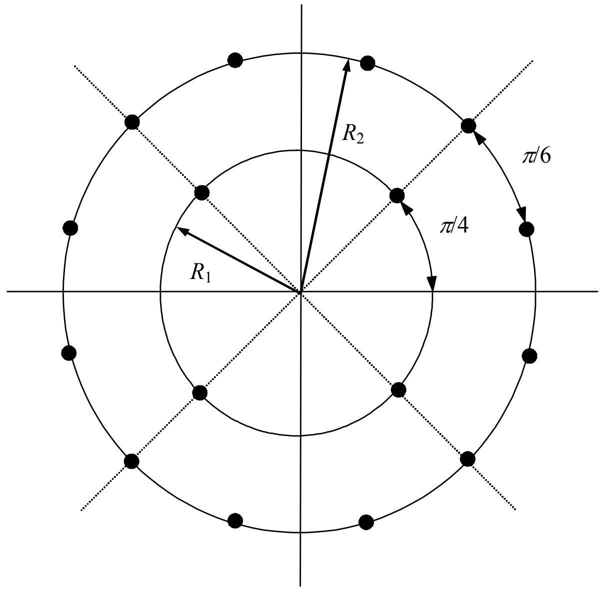


Performance of different FEC schemes (BPSK/AWGN)

Signal model

- ***Modulation schemes***
  - PSK (spectral efficiency)
  - QAM (nonlinear effects: TWT)
  - APSK (DVB-S2 standard)
  - CPM (MSK, GMSK)
- ***Distortions***
  - Additive white Gaussian noise (AWGN)
- ***Transmission parameters***
  - Carrier frequency offset: Δf
  - Carrier phase: θ
  - Symbol period: T
  - Symbol timing: τ

Signal model



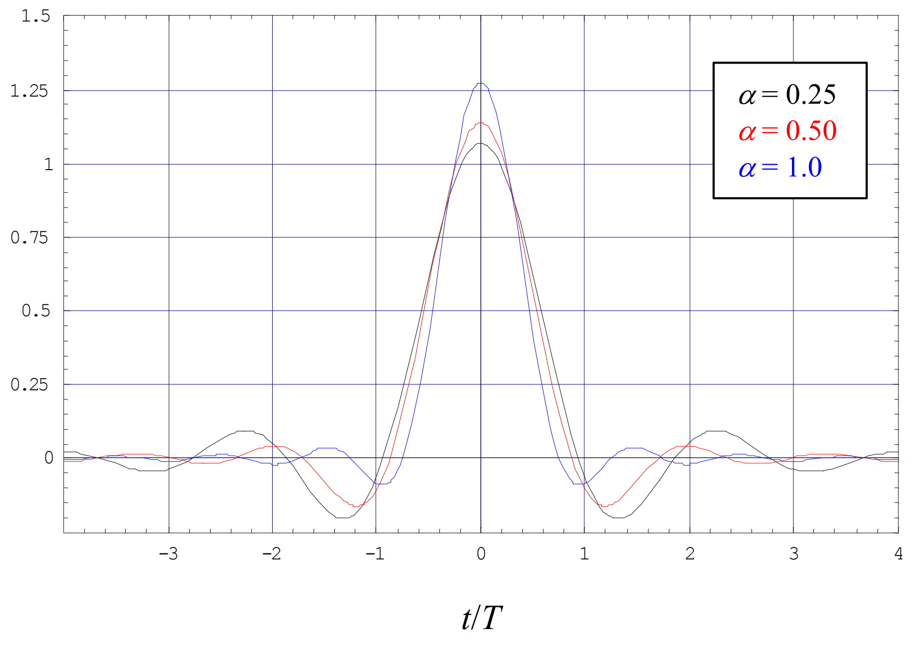
Symbol constellation for 16-APSK

Signal model

- ***Pulse shaping***
  - Nyquist criteria (no ISI)
  - Root-raised cosines (RRCos): $h(t)$
  - Matched filter: $h^*(-t)$
  - Roll-off factor: α
- ***MF input signal***
  - Parameter vector: $\mathbf{v} = (\Delta f, \theta, \tau)$
  - Additive white Gaussian noise (AWGN): $w(t)$

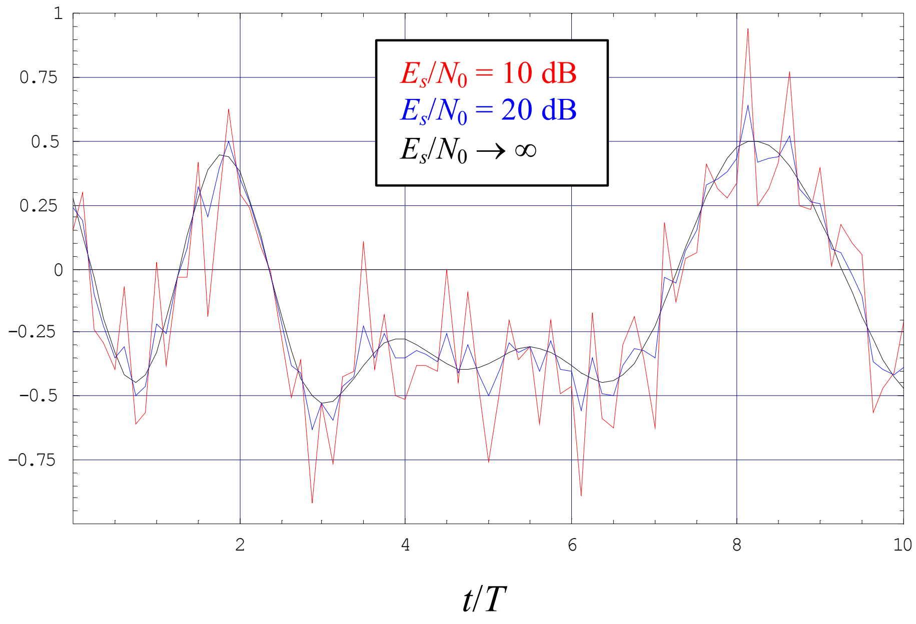
$$r(t,\mathbf{v}) = s(t,\mathbf{v}) + w(t) = e^{j(2\pi\Delta f t+\theta)}\left[\sum_k c_k h(t-kT-\tau)\right] + w(t)$$

Signal model



Impulse response $h(t)$ of different RRCos filters

Signal model



Signal $r(t,\boldsymbol{v})$ at MF input (α = 0.25)

Signal model

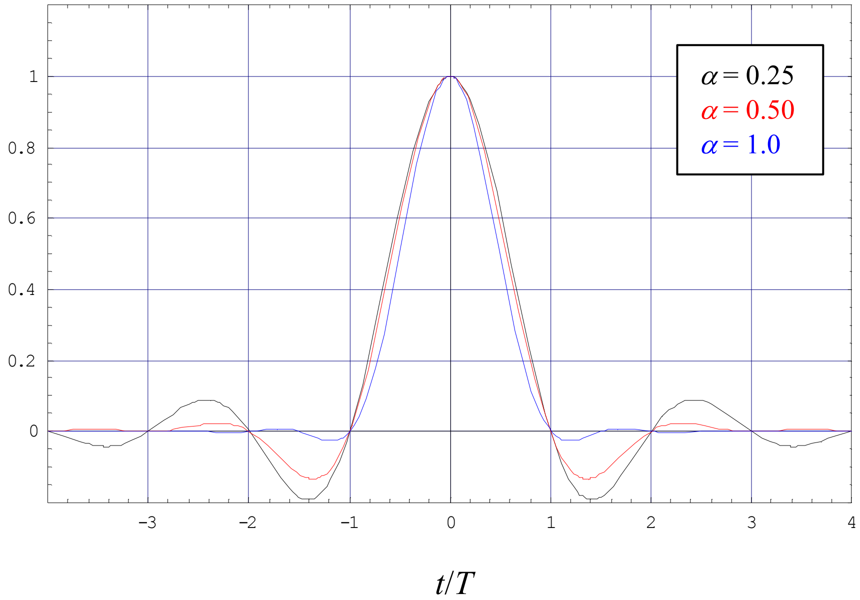
- ***MF output signal***
  - Convolution with MF impulse response $\rightarrow$ RCos
  - Additive non-white Gaussian noise: *n*(*t*)

$$x(t,\mathbf{v}) = r(t,\mathbf{v}) \otimes h^*(-t) = e^{j(2\pi\Delta f t+\theta)}\left[\sum_k c_k g(t-kT-\tau)\right] + n(t)$$

$$g(t) = h(t) \otimes h^*(-t) = \mathrm{sinc}(t/T)\frac{\cos(\alpha\pi t/T)}{1-(2\alpha t/T)^2}$$

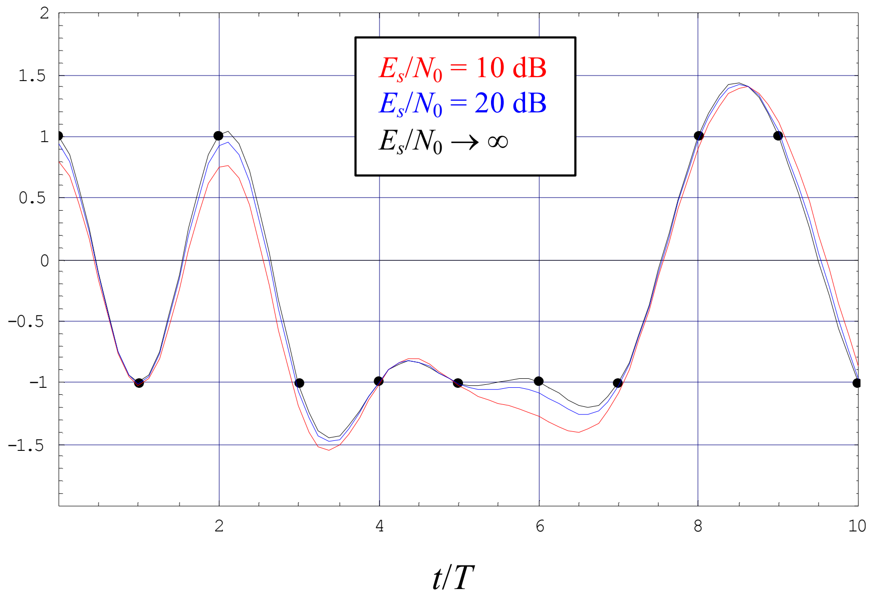
$$n(t) = w(t) \otimes h^*(-t)$$

Signal model



Impulse response *g*(*t*) of different RCos filters

Signal model

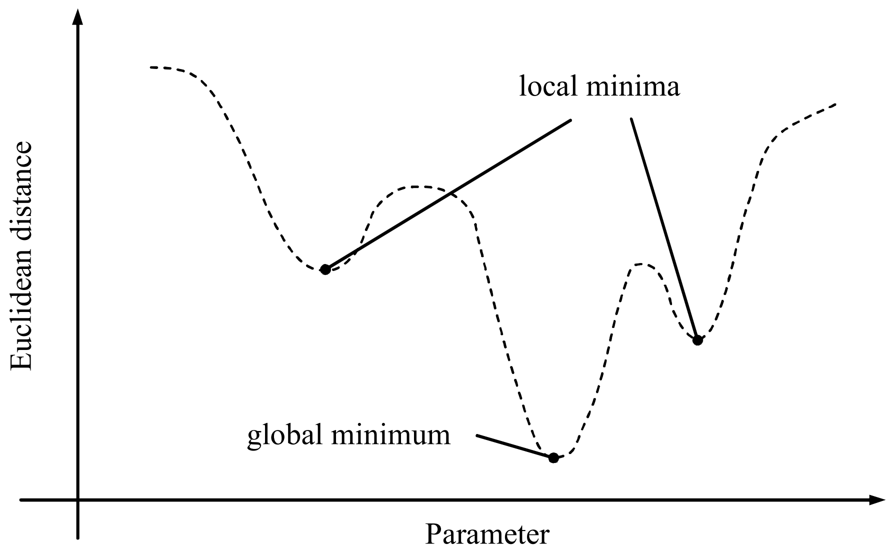


Signal $x(t,\boldsymbol{v})$ at MF output (α = 0.25)

Synchronization in digital receivers

- ***What does synchronization mean***
  - Synchronization in TDMA systems
  - Synchronization as optimization problem
    - Multi-dimensional
    - Nonlinear
    - Stochastic
  - Object function
    - Minimal Euclidean distance between received / replica signal
    - Maximal correlation peak between received / replica signal
  - Synchronization stages
    - Acquisition (initialization)
    - Tracking (controlling)

Synchronization in digital receivers

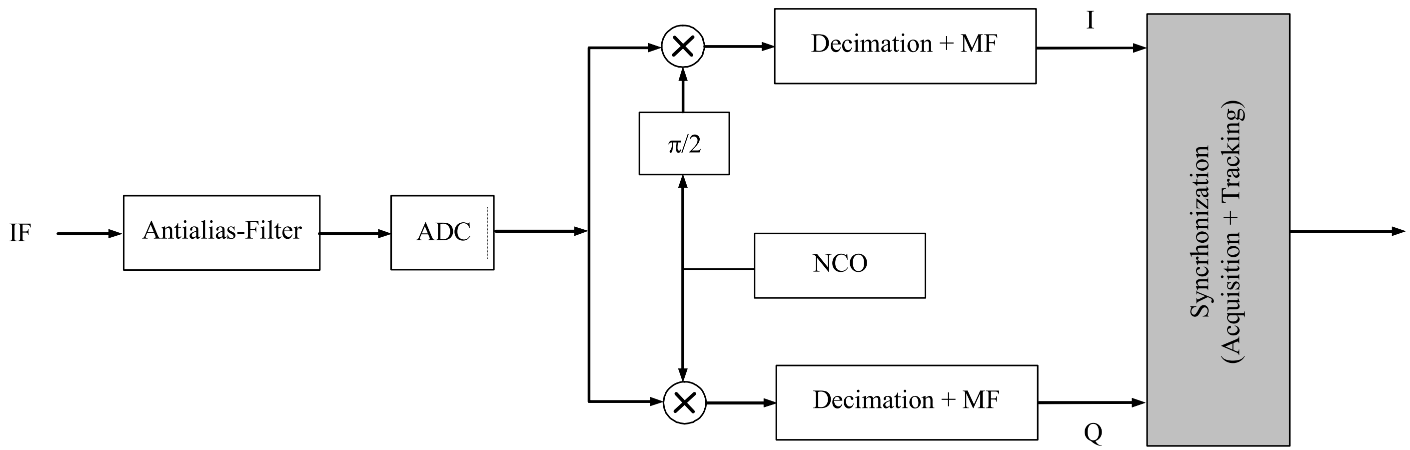


One-dimensional schematic of the synchronization process

Synchronization in digital receivers

- ***Basic receiver architecture***
  - Demodulator
    - IF converter + anti-alias filter
    - AD conversion + mixer stage
    - MF + decimation
  - Synchronization
    - Feedforward algorithms
      - Used for acquisition (data-aided)
      - Specified observation length
      - No stability problems
    - Feedback algorithms
      - Used for tracking (decision-directed, non-data-aided)
      - Probabilistic behavior of the settling time
      - Stability problems (cycle slips, hang-ups)

Synchronization in digital receivers



Basic receiver architecture

Synchronization in digital receivers

- ***Maximum-likelihood principle*** **(1)**
  - Replica signal with vector of trial parameters to be estimated
  - Normal distribution of the probability function
  - Observation length of *L* symbols
  - Maximization procedure

$$f(\tilde{\mathbf{v}}) = C_1 \exp\left[-\frac{C_2}{2\sigma_w^2} \int_0^{LT} \left| r(t,\mathbf{v}) - s(t,\tilde{\mathbf{v}}) \right|^2 dt \right]$$

$$\hat{\mathbf{v}} = \arg\max_{\tilde{\mathbf{v}}} f(\tilde{\mathbf{v}})$$

Synchronization in digital receivers

- ***Maximum-likelihood principle*** **(2)**
  - Log-likelihood function
  - Maximization of the correlation function

$$\Lambda(\tilde{\mathbf{v}}) = \ln f(\tilde{\mathbf{v}}) \sim \mathrm{Re}\left[\int_0^{LT} r(t,\mathbf{v}) s^*(t,\tilde{\mathbf{v}})\, dt\right]$$

$$s(t,\tilde{\mathbf{v}}) = e^{j(2\pi\Delta\tilde{f}t+\tilde{\theta})} \sum_k c_k h(t-kT-\tilde{\tau})$$

$$\Lambda(\tilde{\mathbf{v}}) = \mathrm{Re}\left[\sum_k c_k^* \int_0^{LT} e^{-j(2\pi\Delta\tilde{f}t+\tilde{\theta})}\, r(t,\mathbf{v})\, h^*(t-kT-\tilde{\tau})\, dt\right]$$

Synchronization in digital receivers

- ***Maximum-likelihood principle*** **(3)**
  - Data-aided (DA)
  - Decision-directed (DD)
  - Non-data-aided (NDA)

$$y_k(\tilde{\mathbf{v}}) = \int_0^{LT} e^{-j(2\pi\Delta\tilde{f}t+\tilde{\theta})}\, r(t,\mathbf{v})\, h^*(t-kT-\tilde{\tau})\, dt$$

$$\approx \underbrace{e^{-j(2\pi k\Delta\tilde{f}T+\tilde{\theta})}}_{|\Delta\tilde{f}T| \ll 1} \underbrace{\int_0^{LT} r(t,\mathbf{v})\, h^*(t-kT-\tilde{\tau})\, dt}_{x_k(\tilde{\tau}) := x(kT+\tilde{\tau})}$$

$$\Lambda_{\mathrm{DA}}(\tilde{\mathbf{v}}) = \mathrm{Re}\left[\sum_k c_k^* y_k(\tilde{\mathbf{v}})\right]$$

$$\Lambda_{\mathrm{DD}}(\tilde{\mathbf{v}}) = \mathrm{Re}\left[\sum_k \hat{c}_k^* y_k(\tilde{\mathbf{v}})\right]$$

$$\Lambda_{\mathrm{NDA}}(\tilde{\mathbf{v}}) = \mathrm{Re}\left[\sum_k F[y_k(\tilde{\mathbf{v}})]\right]$$

Synchronization in digital receivers

- ***Estimator criteria***
  - Bias (offset)
  - Jitter variance (CRLB)
  - Consistency property
  - Computational complexity

Feedforward algorithms

- ***Carrier phase estimation*** **(1)**
  - Carrier phase constant (observation length: *L* symbols)
  - No frequency error assumed
  - Symbol timing established
  - Modified CRLB

$$\mathrm{MCRLB} = \frac{1}{2L\,E_s/N_0}$$

Feedforward algorithms

- ***Carrier phase estimation*** **(2)**
  - Signal model
  - Symbols c_k known (DA)
  - ML principle applicable
  - DA unambiguity: $|\theta| < \pi$

$$x_k = e^{j\theta} c_k + n_k$$

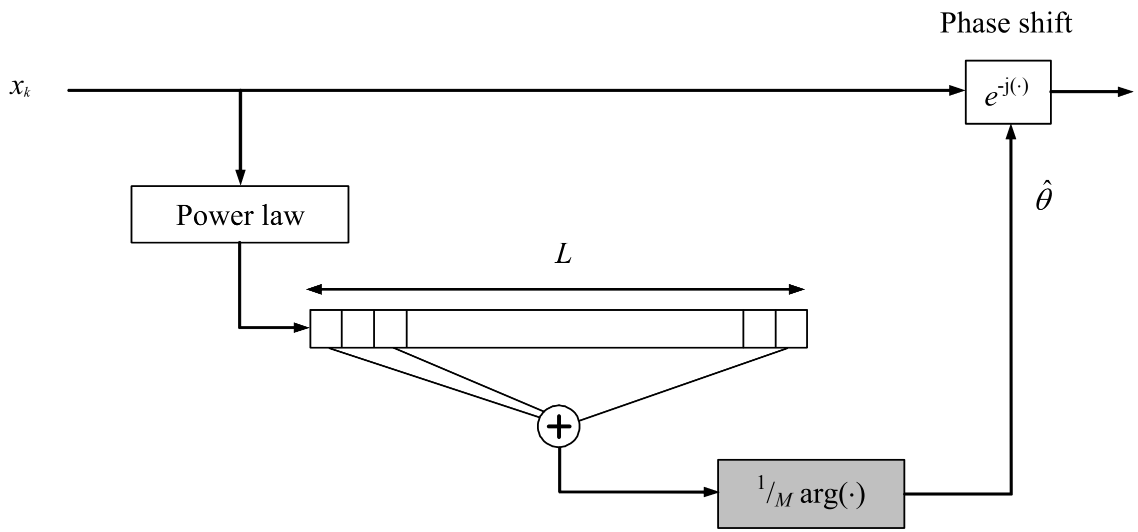
$$\hat{\theta} = \arg\max_{\tilde{\theta}} \mathrm{Re}\left[\sum_k c_k^* x_k e^{-j\tilde{\theta}}\right] = \arg\left(\sum_{k=0}^{L-1} c_k^* x_k\right)$$

Feedforward algorithms

- ***Carrier phase estimation* (3)**
  - Modulation scheme: MPSK
  - Symbols c_k unknown (NDA)
  - Data modulation must be canceled
  - Ad-hoc solution: power-law estimator
  - NDA unambiguity: $|\theta| < \pi / M$

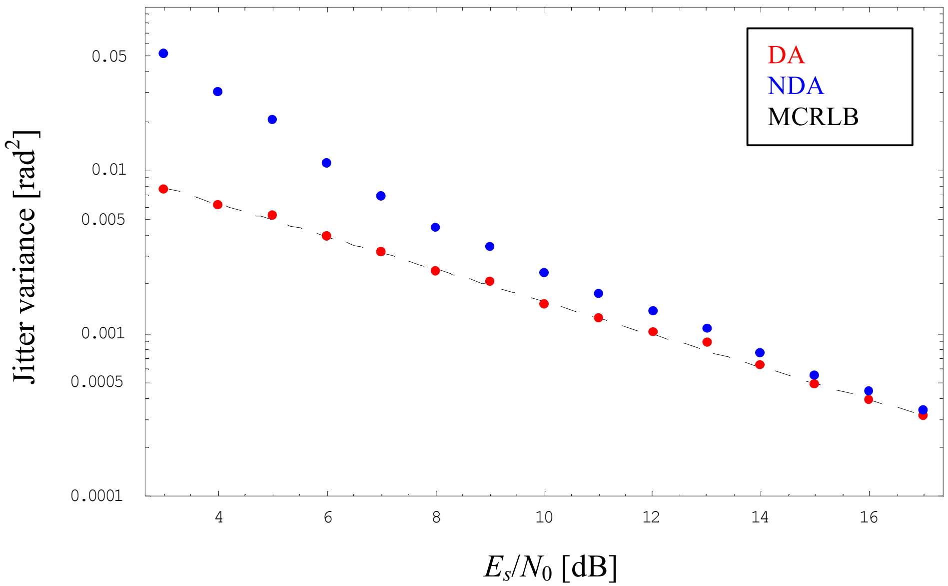
$$\hat{\theta} = \frac{1}{M} \arg\left(\sum_{k=0}^{L-1} x_k^M \right)$$

Feedforward algorithms



NDA feedforward estimator for carrier phase

Feedforward algorithms



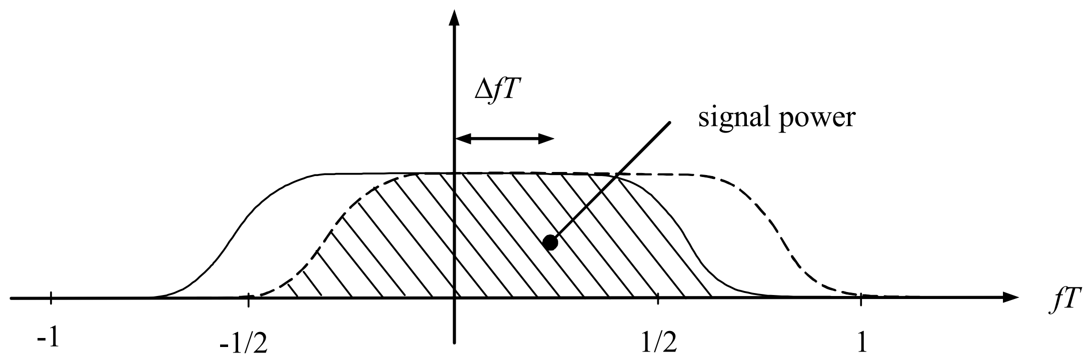
Jitter variance for QPSK (L = 32)

Feedforward algorithms

- ***Carrier frequency estimation*** **(1)**
  - Frequency offset due to Doppler shift and oscillator drifts
  - Signal power affected at MF output
  - Nyquist criterion violated: ISI problems
  - Simplified signal model for analysis ($|\Delta f T| \ll 1$)
  - Symbol timing established

$$x_k = c_k e^{j(2\pi k \Delta f T + \theta)} + n_k$$

Feedforward algorithms



Spectral conditions due to frequency errors

Feedforward algorithms

- ***Carrier frequency estimation*** **(2)**
  - Maximum-likelihood (ML) solution
  - Data-aided estimation
  - Modified CRLB

$$(\Delta\hat{f},\hat{\theta}) = \arg\max_{\Delta\tilde{f},\tilde{\theta}} \mathrm{Re}\left[\sum_k c_k^* x_k e^{-j(2\pi k\Delta\tilde{f}T+\tilde{\theta})}\right]$$

$$\mathrm{MCRLB} = \frac{3}{2\pi^2}\frac{1}{L(L^2-1)E_s/N_0}$$

Feedforward algorithms

- ***Carrier frequency estimation* (3)**
  - ML too complex (searching)
  - Application of DFT (FFT) algorithms: Rife-Boorstyn
  - Autocorrelation principle: Luise-Reggiannini, Mengali-Morelli
  - Main figures of merit
    - Operational range
    - Jitter variance
    - Threshold effect
    - Computational complexity

Feedforward algorithms

- ***Estimation of the symbol timing*** **(1)**
  - Knowledge of carrier frequency/phase irrelevant
  - Data symbols c_k unknown (NDA)
  - ML solution too complex $\rightarrow$ Oerder-Meyr algorithm
  - Modified CRLB

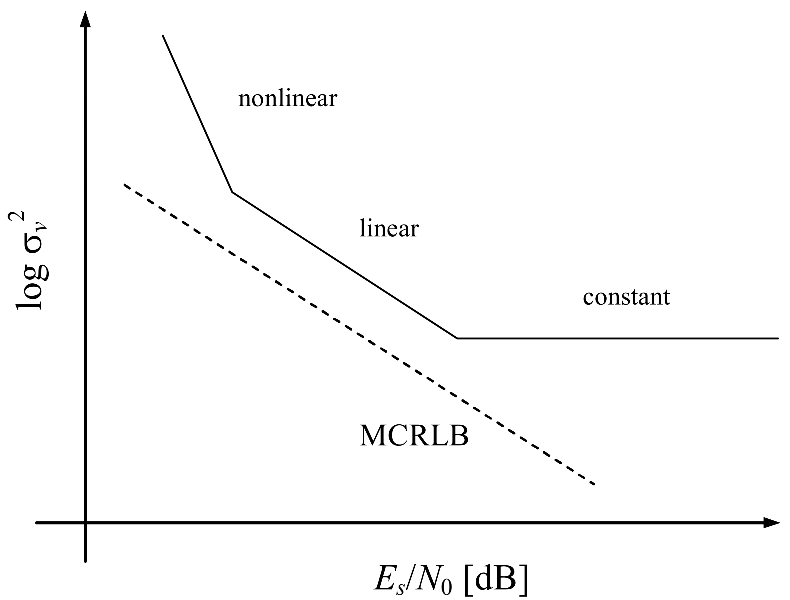
$$\text{MCRLB} = \frac{1}{-2L\, T^2 \ddot{g}(0)\, E_s / N_0}$$

$$-T^2 \ddot{g}(0) = \tfrac{1}{3}\pi^2 (1+3\alpha^2) - 8\alpha^2$$

Feedback algorithms

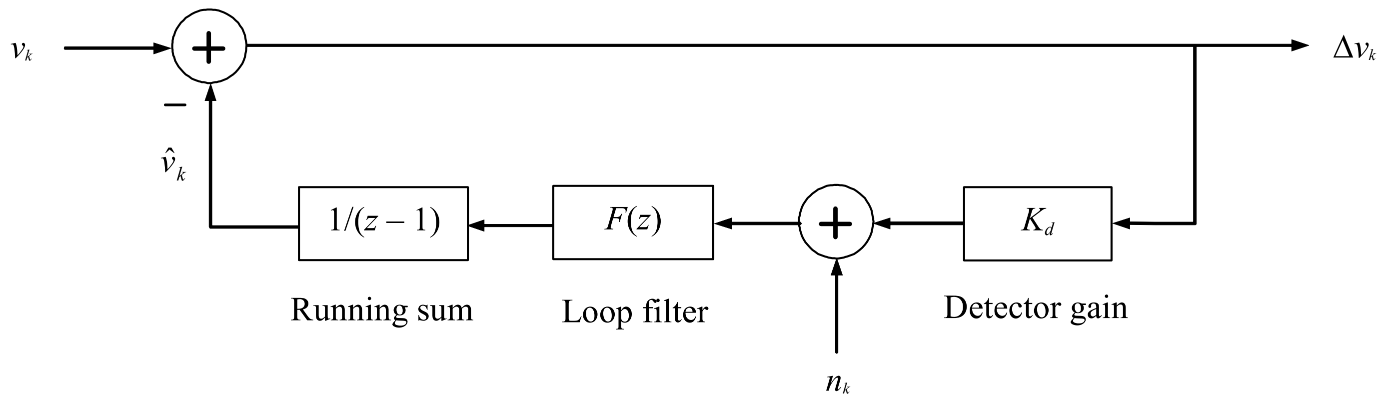
- ***Tracking loops***
  - Carrier phase recovery
  - Symbol timing recovery
  - Main figures of merit
    - Open-loop (detector) characteristic (S-curve)
    - Jitter variance
      - Linear range: ~ MCRLB
      - Constant range: self-noise (ISI)
      - Nonlinear range: threshold effect (DD, NDA)
  - Linearization
    - Loop filter: order of model
    - Slope of S-curve in the stable equilibrium point: detector gain

Feedback algorithms



Jitter variance for feedback recovery loops

Feedback algorithms



Linearized tracker model

Feedback algorithms

- ***Digital recovery loops* (1)**
  - First-order loops
    - $F(z) = K_F$
    - Offset problems if input affected by a drift
    - Loop gain $K_0 = K_F K_d$
    - Transfer function $H_0(z)$

$$H_0(z) = \frac{\Delta v(z)}{v(z)} = \frac{z-1}{z-1+K_0}$$

Feedback algorithms

- ***Digital recovery loops* (2)**
  - Second-order loops
    - $F(z) = a + b / (z - 1)$
    - No offset problems if input affected by a drift
    - Transfer function $H_0(z)$

$$H_0(z) = \frac{(z-1)^2}{(z-1)^2 + aK_d(z-1) + bK_d}$$

Feedback algorithms

- ***Digital recovery loops*** **(3)**
  - Nonlinear effects: hang-ups, cycle slips
  - One-sided equivalent noise bandwidth $B_L T$
  - Transfer function for loop noise: $H_n(z) = 1 - H_0(z)$
  - Jitter variance ~ $B_L T$

$$B_L T = \int_0^{1/2} | H_n(e^{j2\pi fT}) |^2 \, d(fT)$$

Feedback algorithms

- ***Carrier phase recovery*** **(1)**
  - Modified CRLB: $2L \rightarrow 1 / B_L T$
  - DA signal model
    - Symbols c_k known
    - No carrier frequency offset
    - Symbol timing established (no oversampling)

$$x_k = c_k e^{j\theta} + n_k$$

Feedback algorithms

- ***Carrier phase recovery* (2)**
  - ML detector
    - DA log-likelihood function
    - Error signal u_k
    - Detector characteristic (S-curve)

$$\Lambda_{\mathrm{DA}}(\tilde{\theta}) = \mathrm{Re}\left[\sum_k c_k^* x_k e^{-j\tilde{\theta}}\right]$$

$$u_k = \mathrm{Im}[c_k^* x_k\, e^{-j\hat{\theta}}] = \mathrm{Im}[c_k^* y_k]$$

$$S_{\mathrm{DA}}(\theta) = \mathrm{E}\{u_k \mid \hat{\theta} = 0\} = \mathrm{E}\{\mathrm{Im}[c_k^*(c_k e^{j\theta} + n_k)]\} = \sin\theta,\ |\theta| < \pi$$

Feedback algorithms

- ***Carrier phase recovery* (3)**
  - DD detector: $c_k \to$ decisions
  - NDA detector: nonlinearity (power law)
  - Unambiguity: $|\theta| < \pi / M$

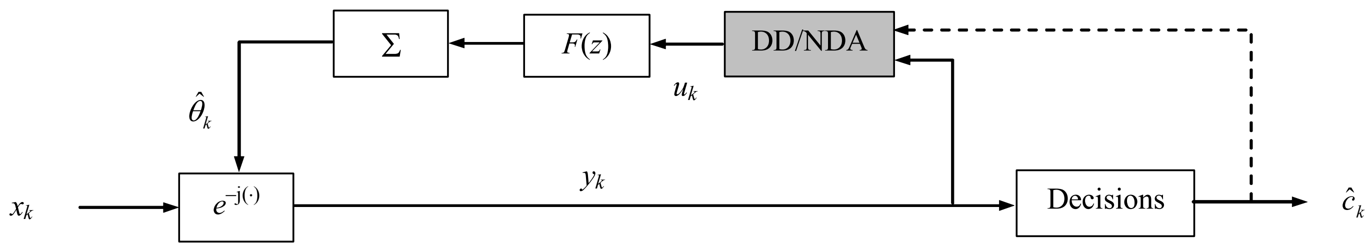
$$u_k = \mathrm{Im}[\hat{c}_k^* y_k]$$

$$\hat{c}_k \to c_k : \; S_{\mathrm{DD}}(\theta) \to S_{\mathrm{DA}}(\theta), \; |\theta| < \frac{\pi}{M}$$

$$u_k = \frac{1}{M} \mathrm{Im}[y_k^M]$$

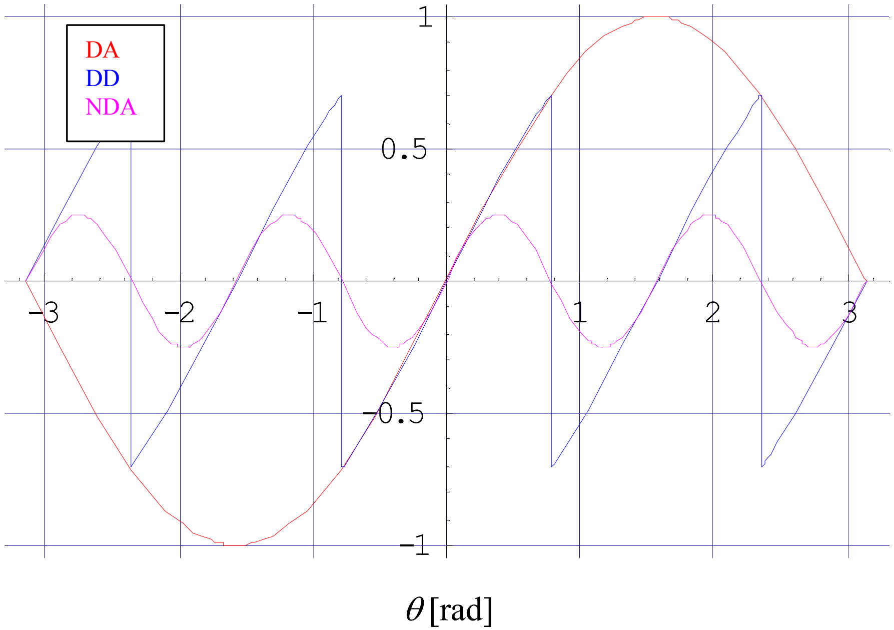
$$S_{\mathrm{NDA}}(\theta) = \mathrm{E}\{u_k \,|\, \hat{\theta} = 0\} = \frac{1}{M} \mathrm{E}\{\mathrm{Im}[(c_k e^{j\theta} + n_k)^M]\} = \frac{1}{M} \sin(M\theta), \, |\theta| < \frac{\pi}{M}$$

Feedback algorithms



Feedback loop for carrier phase recovery

Feedback algorithms



Detector characteristic for QPSK

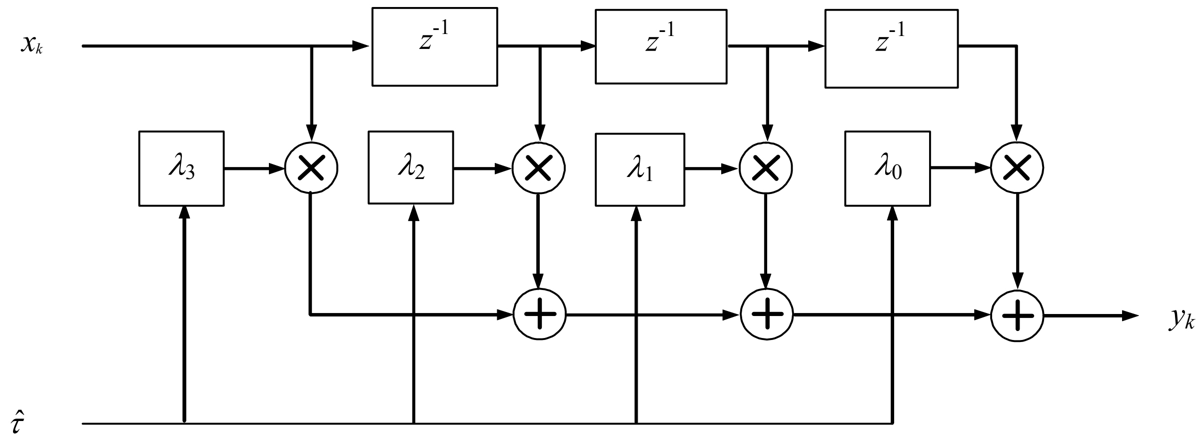
Feedback algorithms

- ***Symbol timing recovery* (1)**
  - Modified CRLB: $2L \to 1 / B_L T$
  - Interpolation
    - Asynchronous (equidistant) sampling
    - Polynomial (Lagrange) interpolation

$$y_k = \sum_{i=0}^{n-1} \lambda_i x_{k+i}$$

$$\lambda_i = \prod_{\substack{k=0 \\ k \neq i}}^{n-1} \frac{\hat{\tau} - t_k}{t_i - t_k} = \prod_{\substack{k=0 \\ k \neq i}}^{n-1} \frac{\hat{\varepsilon} - k}{i - k}$$

Feedback algorithms



Cubic Lagrange interpolator (n = 4)

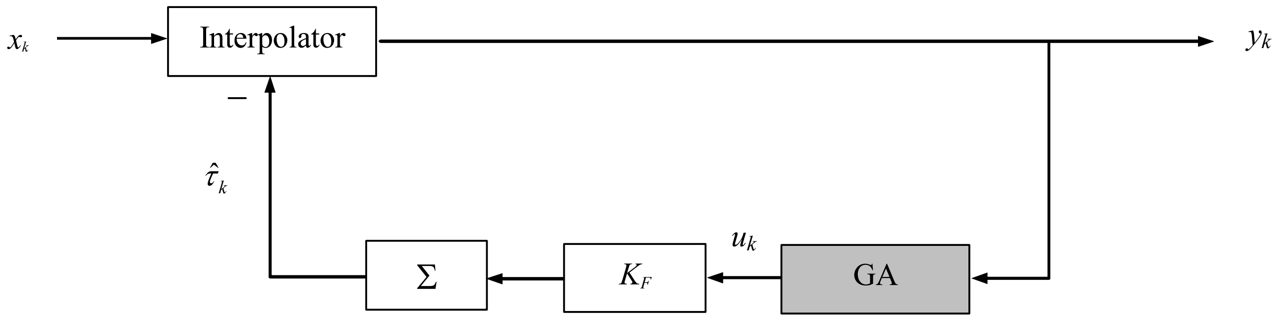
Feedback algorithms

- ***Symbol timing recovery* (2)**
  - NDA detector
    - ML solution too complex
    - NDA and carrier-blind solution
    - Gardner (GA) algorithm
    - S-curve

$$u_k = \mathrm{Re}[(x_k - x_{k-1})x^*_{k-1/2}]$$

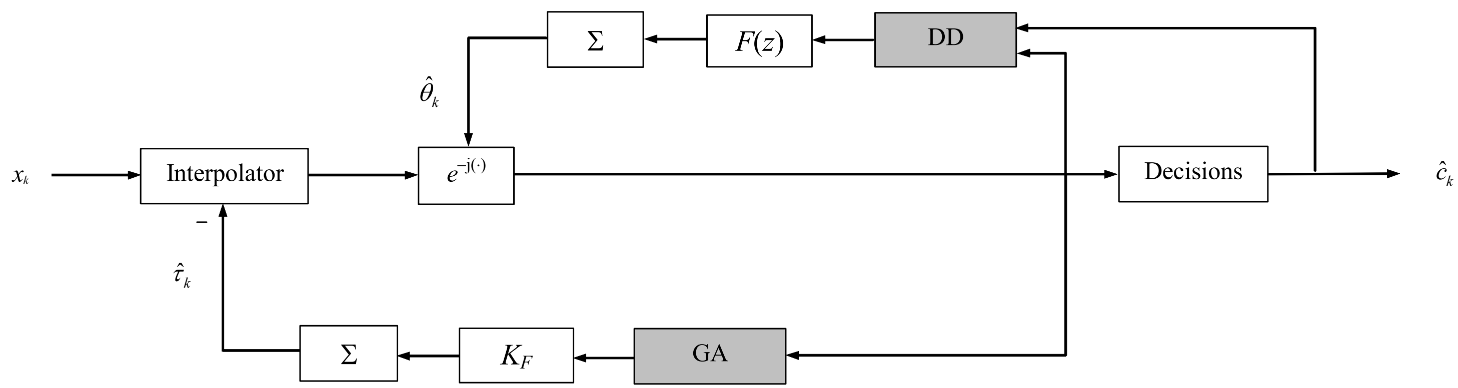
$$S(\varepsilon) = \frac{\sin(\pi\alpha/2)}{\pi(1-\alpha^2/4)}\sin(2\pi\varepsilon)$$

Feedback algorithms



Symbol timing recovery with first-order loops

Feedback algorithms



Joint recovery of carrier phase and symbol timing

Research areas

- ***Satellite monitoring***
  - Detection of modulation schemes
  - Estimation of symbol period and roll-off factor
  - SNR estimation
- ***Joint synchronization and decoding***
  - Parameter estimation + data detection (FEC)
  - Turbo synchronization
    - Maximum-likelihood approach (complexity)
    - Expectation-maximization solution (performance/complexity tradeoff)
    - Per-survivor processing (low complexity)
- ***Fading channels***
  - SNR estimation (ACM systems)
  - Fading parameters
- ***PLL models***
  - Nonlinear-stochastic differential equations
  - Acquisition behavior
  - Phase noise

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