Advanced Modulation/Synchronization/Coding Schemes

Parameter Estimation and Synchronization in Digital Receivers

Dipl.-Ing. Dr. techn. Wilfried Gappmair

Institute of Communication Networks and Satellite Communications
Graz University of Technology, Austria
Contents

- **Introduction**
  - Motivation and background

- **Signal model**
  - Modulation schemes
  - Distortions
  - Transmission parameters
  - Pulse shaping and matched filter

- **Synchronization in digital receivers**
  - What does synchronization mean
  - Basic receiver architecture
  - Maximum-likelihood principle
  - Estimation criteria

- **Feedforward algorithms**
  - Carrier phase estimation
  - Carrier frequency estimation
  - Symbol timing estimation

- **Feedback algorithms**
  - Digital recovery loops
  - Carrier phase recovery
  - Symbol timing recovery

- **Research areas**
Introduction

- **Turbo codes**
  - Berrou et al. (ICC’93)
  - Shannon bound (1948)
  - Computational complexity

- **Demodulator**
  - Synchronization with PLL as standard solution
    - Continuous vs. burst mode
    - Impact of phase noise (small/large bandwidths)
  - Challenging operational conditions: very low SNR values, burst mode
  - Major parameters to be estimated: carrier frequency/phase, symbol timing
Introduction

Basic receiver architecture
Introduction

Performance of different FEC schemes (BPSK/AWGN)
Signal model

- **Modulation schemes**
  - PSK (spectral efficiency)
  - QAM (nonlinear effects: TWT)
  - APSK (DVB-S2 standard)
  - CPM (MSK, GMSK)

- **Distortions**
  - Additive white Gaussian noise (AWGN)

- **Transmission parameters**
  - Carrier frequency offset: $\Delta f$
  - Carrier phase: $\theta$
  - Symbol period: $T$
  - Symbol timing: $\tau$
Signal model

Symbol constellation for 16-APSK
Signal model

- **Pulse shaping**
  - Nyquist criteria (no ISI)
  - Root-raised cosines (RRCos): \( h(t) \)
  - Matched filter: \( h^*(-t) \)
  - Roll-off factor: \( \alpha \)

- **MF input signal**
  - Parameter vector: \( \mathbf{v} = (\Delta f, \theta, \tau) \)
  - Additive white Gaussian noise (AWGN): \( w(t) \)

\[
r(t, \mathbf{v}) = s(t, \mathbf{v}) + w(t) = e^{j(2\pi\Delta f + \theta)} \left[ \sum_k c_k h(t - kT - \tau) \right] + w(t)
\]
Signal model

Impulse response $h(t)$ of different RRCos filters
Signal model

Signal $r(t, \nu)$ at MF input ($\alpha = 0.25$)

$E_s/N_0 = 10 \text{ dB}$
$E_s/N_0 = 20 \text{ dB}$
$E_s/N_0 \to \infty$
Signal model

- **MF output signal**
  - Convolution with MF impulse response → RCos
  - Additive non-white Gaussian noise: \( n(t) \)

\[
x(t, \mathbf{v}) = r(t, \mathbf{v}) \otimes h^*(-t) = e^{j(2\pi f_1 t + \theta)} \left[ \sum_k c_k g(t - kT - \tau) \right] + n(t)
\]

\[
g(t) = h(t) \otimes h^*(-t) = \text{sinc}(t/T) \frac{\cos(\alpha \pi t/T)}{1 - (2\alpha t/T)^2}
\]

\[
n(t) = w(t) \otimes h^*(-t)
\]
Signal model

Impulse response $g(t)$ of different RCos filters
Signal model

Signal $x(t, \nu)$ at MF output ($\alpha = 0.25$)
Synchronization in digital receivers

- *What does synchronization mean*
  - Synchronization in TDMA systems
  - Synchronization as optimization problem
    - Multi-dimensional
    - Nonlinear
    - Stochastic
  - Object function
    - Minimal Euclidean distance between received / replica signal
    - Maximal correlation peak between received / replica signal
  - Synchronization stages
    - Acquisition (initialization)
    - Tracking (controlling)
Synchronization in digital receivers

One-dimensional schematic of the synchronization process
Synchronization in digital receivers

- **Basic receiver architecture**
  - Demodulator
    - IF converter + anti-alias filter
    - AD conversion + mixer stage
    - MF + decimation
  - Synchronization
    - Feedforward algorithms
      - Used for acquisition (data-aided)
      - Specified observation length
      - No stability problems
    - Feedback algorithms
      - Used for tracking (decision-directed, non-data-aided)
      - Probabilistic behavior of the settling time
      - Stability problems (cycle slips, hang-ups)
Synchronization in digital receivers

Basic receiver architecture
Synchronization in digital receivers

- **Maximum-likelihood principle (1)**
  - Replica signal with vector of trial parameters to be estimated
  - Normal distribution of the probability function
  - Observation length of $L$ symbols
  - Maximization procedure

\[
f(\tilde{v}) = C_1 \exp \left[ - \frac{C_2}{2\sigma_w^2} \int_0^L \left| r(t, v) - s(t, \tilde{v}) \right|^2 dt \right]
\]

\[
\hat{v} = \arg \max_{\tilde{v}} f(\tilde{v})
\]
Synchronization in digital receivers

- **Maximum-likelihood principle (2)**
  - Log-likelihood function
  - Maximization of the correlation function

\[
\Lambda(\tilde{\nu}) = \ln f(\tilde{\nu}) \sim \text{Re} \left[ \int_0^{LT} r(t, \nu) s^*(t, \tilde{\nu}) \, dt \right]
\]

\[
s(t, \tilde{\nu}) = e^{j(2\pi\tilde{\nu}t + \tilde{\theta})} \sum_k c_k h(t - kT - \tilde{\tau})
\]

\[
\Lambda(\tilde{\nu}) = \text{Re} \left[ \sum_k c_k^* \int_0^{LT} e^{-j(2\pi\tilde{\nu}t + \tilde{\theta})} r(t, \nu) h^*(t - kT - \tilde{\tau}) \, dt \right]
\]
Synchronization in digital receivers

- **Maximum-likelihood principle (3)**
  - Data-aided (DA)
  - Decision-directed (DD)
  - Non-data-aided (NDA)

\[
y_k(\bar{v}) = \int_0^{LT} e^{-j(2\pi\bar{y} + \bar{\theta})} r(t, \nu) h^*(t - kT - \bar{\tau}) \, dt
\]

\[
\approx e^{-j(2\pi k \Delta T + \bar{\theta})} \int_0^{LT} r(t, \nu) h^*(t - kT - \bar{\tau}) \, dt
\]

\[
\Delta_{DA}(\bar{v}) = \text{Re} \left[ \sum_k \hat{c}_k^* y_k(\bar{v}) \right]
\]

\[
\Delta_{DD}(\bar{v}) = \text{Re} \left[ \sum_k \hat{c}_k^* y_k(\bar{v}) \right]
\]

\[
\Delta_{NDA}(\bar{v}) = \text{Re} \left[ \sum_k F[y_k(\bar{v})] \right]
\]
Synchronization in digital receivers

- **Estimator criteria**
  - Bias (offset)
  - Jitter variance (CRLB)
  - Consistency property
  - Computational complexity
Feedforward algorithms

- **Carrier phase estimation** (1)
  - Carrier phase constant (observation length: $L$ symbols)
  - No frequency error assumed
  - Symbol timing established
  - Modified CRLB

\[
MCRLB = \frac{1}{2LE_s/N_0}
\]
Feedforward algorithms

- **Carrier phase estimation (2)**
  - Signal model
  - Symbols $c_k$ known (DA)
  - ML principle applicable
  - DA unambiguity: $|\theta| < \pi$

\[
x_k = e^{j\theta} c_k + n_k
\]

\[
\hat{\theta} = \arg \max_{\theta} \Re \left[ \sum_k c_k^* x_k e^{-j\theta} \right] = \arg \left( \sum_{k=0}^{l-1} c_k^* x_k \right)
\]
Feedforward algorithms

- **Carrier phase estimation** (3)
  - Modulation scheme: MPSK
  - Symbols $c_k$ unknown (NDA)
  - Data modulation must be canceled
  - Ad-hoc solution: power-law estimator
  - NDA unambiguity: $|\theta| < \pi / M$

$$\hat{\theta} = \frac{1}{M} \arg \left( \sum_{k=0}^{L-1} x_k^M \right)$$
Feedforward algorithms

NDA feedforward estimator for carrier phase
Feedforward algorithms

Jitter variance for QPSK ($L = 32$)
Feedforward algorithms

- **Carrier frequency estimation** (1)
  - Frequency offset due to Doppler shift and oscillator drifts
  - Signal power affected at MF output
  - Nyquist criterion violated: ISI problems
  - Simplified signal model for analysis ($|ΔfT| ≪ 1$ )
  - Symbol timing established

\[ x_k = c_k e^{j(2πkΔfT + θ)} + n_k \]
Feedforward algorithms

Spectral conditions due to frequency errors
Feedforward algorithms

- **Carrier frequency estimation (2)**
  - Maximum-likelihood (ML) solution
  - Data-aided estimation
  - Modified CRLB

\[
(\Delta \hat{f}, \hat{\theta}) = \arg \max_{\Delta \hat{f}, \hat{\theta}} \text{Re} \left[ \sum_k c_k^* x_k e^{-j(2\pi k \Delta \hat{f} T + \hat{\theta})} \right]
\]

MCRLB = \frac{3}{2 \pi^2} \frac{1}{L(L^2 - 1) E_s / N_0}
Feedforward algorithms

- **Carrier frequency estimation** (3)
  - ML too complex (searching)
  - Application of DFT (FFT) algorithms: Rife-Boorstyn
  - Autocorrelation principle: Luise-Reggiannini, Mengali-Morelli
  - Main figures of merit
    - Operational range
    - Jitter variance
    - Threshold effect
    - Computational complexity
Feedforward algorithms

- **Estimation of the symbol timing** (1)
  - Knowledge of carrier frequency/phase irrelevant
  - Data symbols $c_k$ unknown (NDA)
  - ML solution too complex $\rightarrow$ Oerder-Meyr algorithm
  - Modified CRLB

\[
\text{MCRLB} = \frac{1}{-2L T^2 \ddot{g}(0) E_s / N_0}
\]

\[
- T^2 \ddot{g}(0) = \frac{1}{3} \pi^2 (1 + 3\alpha^2) - 8\alpha^2
\]
Feedback algorithms

- **Tracking loops**
  - Carrier phase recovery
  - Symbol timing recovery
  - Main figures of merit
    - Open-loop (detector) characteristic (S-curve)
    - Jitter variance
      - Linear range: \( \sim \) MCRLB
      - Constant range: self-noise (ISI)
      - Nonlinear range: threshold effect (DD, NDA)
  - Linearization
    - Loop filter: order of model
    - Slope of S-curve in the stable equilibrium point: detector gain
Feedback algorithms

Jitter variance for feedback recovery loops
Feedback algorithms

Linearized tracker model

\[ F(z) = \frac{1}{z-1} \]

Detector gain

Running sum

Loop filter

\[ n_i \]

\[ \hat{v}_k \]
Feedback algorithms

- Digital recovery loops (1)
  - First-order loops
    - $F(z) = K_F$
    - Offset problems if input affected by a drift
    - Loop gain $K_0 = K_F K_d$
    - Transfer function $H_0(z)$

\[ H_0(z) = \frac{\Delta v(z)}{v(z)} = \frac{z - 1}{z - 1 + K_0} \]
Feedback algorithms

- **Digital recovery loops (2)**
  - Second-order loops
    - $F(z) = a + b / (z - 1)$
    - No offset problems if input affected by a drift
    - Transfer function $H_0(z)$

$$H_0(z) = \frac{(z - 1)^2}{(z - 1)^2 + aK_d (z - 1) + bK_d}$$
Feedback algorithms

- **Digital recovery loops (3)**
  - Nonlinear effects: hang-ups, cycle slips
  - One-sided equivalent noise bandwidth $B_L T$
  - Transfer function for loop noise: $H_n(z) = 1 - H_0(z)$
  - Jitter variance $\sim B_L T$

\[
B_L T = \int_0^{1/2} |H_n(e^{j2\pi fT})|^2 \, d(fT)
\]
Feedback algorithms

- **Carrier phase recovery (1)**
  - Modified CRLB: $2L \rightarrow 1 / B_L T$
  - DA signal model
    - Symbols $c_k$ known
    - No carrier frequency offset
    - Symbol timing established (no oversampling)

$$x_k = c_k e^{j\theta} + n_k$$
Feedback algorithms

- **Carrier phase recovery (2)**
  - ML detector
    - DA log-likelihood function
    - Error signal $u_k$
    - Detector characteristic (S-curve)

\[
\Lambda_{DA}(\tilde{\theta}) = \text{Re} \left[ \sum_k c_k^* x_k e^{-j\tilde{\theta}} \right]
\]

\[
u_k = \text{Im}[c_k^* x_k e^{-j\tilde{\theta}}] = \text{Im}[c_k^* y_k]
\]

\[
S_{DA}(\theta) = \mathbb{E}\{u_k \mid \hat{\theta} = 0\} = \mathbb{E}\{\text{Im}[c_k^*(c_k e^{j\theta} + n_k)]\} = \sin \theta, \ |\theta| < \pi
\]
Feedback algorithms

- **Carrier phase recovery** (3)
  - DD detector: $c_k \rightarrow$ decisions
  - NDA detector: nonlinearity (power law)
  - Unambiguity: $|\theta| < \pi / M$

\[
\hat{c}_k \rightarrow c_k : S_{DD}(\theta) \rightarrow S_{DA}(\theta), \quad |\theta| < \frac{\pi}{M}
\]
\[
u_k = \frac{1}{M} \text{Im} [y_k^M]
\]
\[
S_{ND}(\theta) = \mathbb{E} \{ u_k | \hat{\theta} = 0 \} = \frac{1}{M} \mathbb{E} \{ \text{Im} [ (c_k e^{i\theta} + n_k)^M ] \} = \frac{1}{M} \sin(M\theta), \quad |\theta| < \frac{\pi}{M}
\]
Feedback algorithms

Feedback loop for carrier phase recovery
Feedback algorithms

Detector characteristic for QPSK
Feedback algorithms

- **Symbol timing recovery** (1)
  - Modified CRLB: $\frac{2L}{1 / B_L T}$
  - Interpolation
    - Asynchronous (equidistant) sampling
    - Polynomial (Lagrange) interpolation

\[
y_k = \sum_{i=0}^{n-1} \lambda_i x_{k+i}
\]

\[
\lambda_i = \prod_{\substack{k=0 \atop k \neq i}}^{n-1} \frac{\hat{\tau} - t_k}{t_i - t_k} = \prod_{\substack{k=0 \atop k \neq i}}^{n-1} \frac{\hat{\varepsilon} - k}{i - k}
\]
Feedback algorithms

Cubic Lagrange interpolator \((n = 4)\)
Feedback algorithms

- **Symbol timing recovery (2)**
  - NDA detector
    - ML solution too complex
    - NDA and carrier-blind solution
    - Gardner (GA) algorithm
    - S-curve

\[
u_k = \text{Re}[(x_k - x_{k-1})x_{k-1/2}^*]
\]

\[
S(\varepsilon) = \frac{\sin(\pi\alpha/2)}{\pi(1 - \alpha^2/4)} \sin(2\pi\varepsilon)
\]
Feedback algorithms

Symbol timing recovery with first-order loops
Feedback algorithms

Joint recovery of carrier phase and symbol timing
Research areas

- **Satellite monitoring**
  - Detection of modulation schemes
  - Estimation of symbol period and roll-off factor
  - SNR estimation

- **Joint synchronization and decoding**
  - Parameter estimation + data detection (FEC)
  - Turbo synchronization
    - Maximum-likelihood approach (complexity)
    - Expectation-maximization solution (performance/complexity tradeoff)
    - Per-survivor processing (low complexity)

- **Fading channels**
  - SNR estimation (ACM systems)
  - Fading parameters

- **PLL models**
  - Nonlinear-stochastic differential equations
  - Acquisition behavior
  - Phase noise
References